A CONTINUOUS-DISCONTINUOUS MODEL FOR SOFTENING AND CRACKING BASED ON NON-LOCAL GRADIENT ELASTICITY

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Summary. A continuous-discontinuous model entirely based on displacements is presented. Crack initiation and propagation is modelled in a smeared way, and regularised by means of a gradient-enriched displacement field, whereas the final stages of failure are represented with a discrete crack approach (X-FEM), with actual discontinuities in the two displacement fields.

1 INTRODUCTION

A generic non-local approach to regularise strain-softening continua is presented. The key idea is to introduce the gradient-type enrichment at the level of displacements [1, 2] (rather than some internal variable), so the model is formulated with two distinct displacement fields: the standard displacements $\mathbf{u}_a$ and the gradient-enriched displacements $\mathbf{u}_g$. In fact, gradient models based on two displacement fields are usual in non-local elasticity, where the goal is to avoid the shortcomings of classical (local) elasticity (i.e. strain singularities in statics, non-dispersive behaviour in dynamics). We show that such a gradient elasticity backbone model can be combined with any standard nonlinear constitutive driver to render a regularised model for softening inelasticity. This generic continuous framework is subsequently enriched with discontinuities in the displacement fields.

2 CONTINUOUS MODEL

The generic expression of the gradient-enriched inelastic constitutive equation is

$$ \sigma(\varepsilon_a, \varepsilon_g) = C : \varepsilon_a - s^{\text{inel}}(\varepsilon_a, \varepsilon_g) $$

where $\varepsilon_a = \nabla \mathbf{u}_a$ is the auxiliary local strain and $\varepsilon_g = \nabla \mathbf{u}_g$ is the gradient-enriched strain. Note that the elastic part of the stress depends only on the local strain $\varepsilon_a$, so the non-local strain $\varepsilon_g$ only appears on the inelastic part of the stress.
In a damage model, the inelastic stress is \( s^{\text{inel}} = \omega(\varepsilon_g)C : \varepsilon_a \), with \( \omega \) the damage parameter, see [1, 2]. For plasticity, one option is to define a model with non-local plastic strain, with the inelastic stress defined as \( s^{\text{inel}} = C : \varepsilon_g^p \) (with \( \varepsilon_g^p \) the plastic part of \( \varepsilon_g \)). Such a model does regularise softening, as illustrated by the biaxial compression test of Figure 1. Neither the deformation pattern, the width of the shear band nor the force-displacement curve (not shown in Figure 1) depend on the finite element mesh.

![Figure 1: Plastic model with non-local plastic strain, biaxial compression test: (a) problem statement; equivalent plastic strain for meshes of (b) 6 \times 12 \text{ elements}; (c) 12 \times 24 \text{ elements}; (d) 24 \times 48 \text{ elements}](image)

However, this model locks at the late stages of softening, see Figure 2(a), and cannot be used to describe complete softening down to zero stress. The reason for this locking response, also exhibited by a similar non-local plastic model of the integral type, is described in [3]. One possible remedy is to formulate the non-local model in terms of a weighted softening variable \( \bar{q} \):

\[
\bar{q} = (1 - m)q_a + mq_g \tag{2}
\]

In Equation (2), \( q_a \) and \( q_g \) are the local and non-local softening variables, computed respectively from strains \( \varepsilon_a \) and \( \varepsilon_g \), and \( m \) is a weight parameter (a typical value is \( m = 2 \), see [3]). With this non-local model, softening is regularised with no locking effects, see Figure 2(b).

### 3 CONTINUOUS-DISCONTINUOUS MODEL

In a second stage, the generic continuous model is enriched with discontinuity interfaces representing cracks. In the fashion of the eXtended Finite Element Method (X-FEM), discontinuous displacement fields are represented as

\[
\mathbf{u}_a(x) = \mathbf{u}^1_a(x) + \mathcal{H}_{\Gamma_d}(x)u^2_a(x) \\
\mathbf{u}_g(x) = \mathbf{u}^1_g(x) + \mathcal{H}_{\Gamma_d}(x)u^2_g(x)
\]

where \( \mathbf{u}^i_a(x) \) and \( \mathbf{u}^i_g(x) \) \((i = 1, 2)\) are continuous fields, and discontinuities along the interface \( \Gamma_d \) are taken care of by the Heaviside function \( \mathcal{H}_{\Gamma_d} \).
Again, we have checked this generic framework first for the case of damage. A damage model is used for the continuum, whereas the crack is described by a cohesive law relating traction to displacement jump. The results for a uniaxial tensile test are shown in Figure 3. The regularising capabilities of the model are clear both from the damage profiles and the force-displacement curves, where the three slopes corresponding to elastic loading, softening in the continuum and crack opening can be seen.
4 CONCLUSIONS

The proposed continuous-discontinuous model is entirely based on displacements. Apart from the usual local displacement field, an additional gradient-enriched field is used. When required, discontinuities are admitted in these two displacement fields.

Using displacements (rather than some model-dependent internal variable) to inject non-locality makes the approach general. It can accommodate various inelastic models (as illustrated here with isotropic damage and von Mises plasticity), both in statics and in dynamics.

It also connects very naturally with gradient elasticity (where the use of enriched displacement fields is motivated by the removal of singularities and the description of wave dispersion).

The extension to a continuous-discontinuous approach is also straightforward. The idea is simply to include discontinuities in the two displacement fields, which can be interpolated by means of the same shape and enrichment functions.

REFERENCES

